Title: Applied Math and Engineering: Algebra 2 and Trig for Engineers

Length of Course: Full Year (2 semesters; 3 trimesters; 4 quarters)

Subject Area: Math ("c")

UC Honors designation: No

CTE Sector: Engineering and Architecture

CTE Pathway: Engineering Design

Grade Level(s): 10-12

Course Overview:

This course allows students to apply Advanced Algebra and Geometry skills contained in the traditional Algebra 2/Trig course to the Engineering Design Process: Requirements, Analysis, Design, Build, Validation. Student groups will represent civil engineering firms who receive a **R**equest **F**or **P**roposal (RFP) from a school district in need of a new 21st Century state-of-the-art high school. Students employ the engineering process to design this high school, considering the parameters and requirements of the district while using math (linear equations and functions, quadratic equations and functions, polynomials, rational and radical functions, exponential and logarithmic functions, statistics and probability, and trigonometry) as a tool to make engineering decisions and complete the projects. Throughout the course, teams will keep an engineering notebook which documents all mathematical calculations, assumptions, notes, preliminary sketches, etc. to provide a "roadmap" of their final design assignment. Project Teams will continuously update their Know/Need To Know Lists Re: the RFP to include new information learned. These revised lists will be used by the teacher as a formative assessment on a recurring basis. The teacher will look for understanding of mathematical concepts as applied to the current engineering process. Upon completion of the course, students will be able to integrate the math topics and concepts with the Engineering Design Process to create a final report and presentation of their high school design aimed at an audience of school board members.

Unit 1: Define the Problem

In this unit students focus on understanding the contents of the RFP, which will require them to gather data to define the needs and requirements of the proposed high school. Students create functions, tables and graphs to analyze the data they collect, in order to understand student demographics, population growth and cost proposals, all information they need to make design decisions. Students will survey potential transfer

students to make statistically driven decisions about school features they will include in their design. Students ultimately understand and use mathematics as a tool that can help current and future engineers arrive at data-based decisions as they work later in the course to design a 21st century high school.

Unit 1: Assignments

Assignment 1

Students will build knowledge of the high school design project by closely examining an RFP provided by the teacher. Students read, annotate and discuss the RFP with the purpose of ultimately having a common class understanding of its components. The RFP asks the design groups to Design a High School that:

- Supports the expected population for the next 10 years
- Meets the cost target of $15M
- Provides a state-of-the-art learning environment
- Offers athletic/activity facilities based on community input
- Incorporates current "green" building technology

By the end of the year at the completion of the project, design teams will provide the following to the School District:

- Presentation showing features of the school
- Final report including the following sections: Requirements, Analysis, Design, Build and Validation
- Optional Scale Model of the proposed high school

Individual students will initial and sign a Project Team Contract to show that they understand and agree to the parameters set forth by the school district. Each Project Team will create a list of "Know" and "Need to Know" items to be used by the teacher to formatively assess the knowledge of the students and provide support throughout the project. This list will be revisited in the middle of the project timeline as a formative assessment.

Assignment 2

Data from local school districts and communities -- including tables of student populations of surrounding schools from the past three years, graphs of student population vs average cost per square footage of the school, as well as extraneous data -- will be provided to students and students will decide on the pertinent data sets for student population projections and growth in order to determine capacity for the high school. Students will apply the statistical concepts of central limit theorem, mean, standard deviation, normal distribution, confidence levels and margin of error to analyze the data. Software will be used by groups to create a spreadsheet to graph important data and generate best fit functions and standard deviations. Students will use the student populations projections from the last three years and their understanding of growth rate to estimate the capacity currently needed by the high school they are designing. This estimate will be included in their spreadsheet with their graphs. The data

from this assignment will be used in conjunction with further data to draft the first section of the final report. All data collected will be kept as part of students' engineering notebooks.

Assignment 3

Students will determine a statistically significant sample size and design a survey. The survey will be used to find the potential number of transfer students to the high school and the features that potential transfer students would want in the high school (i.e., labs, gym, swimming pool, etc). For this assignment, they will use potential numbers of transfer students to revise the needed capacity of the facility and site from Assignment 2. For assignment 3, students will use teacher provided data from the sample survey to estimate a population mean and develop a margin of error. These data evaluations will be recorded in their engineering notebooks.

Assignment 4

Students will use mock survey data provided by the teacher regarding desired features of the high school by the transfer students from Assignment 3 to make decisions about what their design will include. Using the survey results of desired features of potential transfer students, project teams will choose school features based on cost and target student size. For each school feature, students will include a cost function with respect to student body size. Based on their population and school features, students will then have a ballpark figure of the cost of building the school. Students will present the projected size and features of their high school to at least one other Project Team. In their presentations, teams will construct a viable argument for their projected capacity and features, using mathematical and textual evidence and critiquing the reasoning of other groups.

Assignment 5

Project Team groups will generate a draft of the first section of their culminating report. The draft will include all of the mathematical calculations and analysis used to determine the initial considerations of the high school design project. Evidence must be included from the RFP text, the survey and the mathematical models created in Assignments 1-5. Drafts of the first section are used to answer: What is the current capacity of the high school? How much growth capacity did your group account for? How long is the projected timeline for growth? What features did you decide to include in the high school? Why? What is your projected cost?

Unit 2: Analysis

In this unit, students assess data gathered in Unit 1 and update their Know / Need To Know lists. They will analyze different site options based on their topological features and use math (such as modeling site selections using parabolas and deriving piecewise quadratic equations to create outlines) to eventually make decisions regarding site choice. Students explore cost functions and convert measurements using dimensional analysis to compare alternatives for space, power and labor costs. They will need to answer the following questions in the second section of their cumulative final report

based on the individual subsystem requirements given below and include important graphs to describe and summarize their analysis. Students will be provided with basic costs and will need to create an equation to model the cost based on their needs.

- Site Survey - What is the cost to grade a plot of land?
- Building(s) - What is the average cost square foot cost of a classroom building, athletic building, theater, cafeteria, gym, fabrication lab, athletic fields, swim pool?
- Electrical Power - How much power per square ft can a solar panel generate? What is the cost of a solar panel per square foot? What is the typical average power requirement per student?
- Plumbing - What are the number of toilets and sinks/fountains and the associated costs?
- Heating, Ventilation and Air Conditioning (HVAC)- How much heating and air conditioning capacity is required per cubic foot of room and how much does it cost per cubic foot?

Unit 2: Assignments

Assignment 1

Students will analyze multiple sites having contour mounds that can be modeled using parabolas. Students will need to derive the equations for the piecewise quadratic functions to create outlines of the mounds on a coordinate plane. They will need to find the intersection point of the two mounds/parabolas and draw trapezoids and triangles that will approximate the area of the curves. Students will take these areas to approximate the volume of the soil that needs to be removed for construction purposes and analyze the associated costs in order to determine the most economical site selection. The calculations made regarding volume of soil needed for excavation and the costs associated with that project will be recorded in the engineering notebook for teacher review.

Assignment 2

In this assignment, students will convert units of measurements through dimensional analysis. Using the provided average cost/square foot of the various types of buildings (see overview), students will create an equation that models the cost of each type of building. This equation will factor into making decisions during the design process related to final costs. Students use size of site and compare costs associated with multistory building vs 1 story building using stepwise functions as their guide to compare costs of building “up” or “out”. The processes for this assignment will be recorded in team’s engineering notebooks for teacher review.

Assignment 3

Students will research basic Leadership in Energy and Environmental Design (LEED) certification to make design decisions, such as types of lighting (LED, etc.), windows, materials, heating, ventilation and air condition (HVAC) to minimize energy consumption. Using the provided cost/square foot of solar panels and output power of solar panels, students will create an equation that models the cost for power. Students will convert

measurements through dimensional analysis. The equation from this assignment will be recorded in the team's engineering notebook for use in the culminating final report. The report is important because it summarizes the design decisions based on the calculations to maximize energy efficiency and therefore minimize cost.The engineering notebook will be used as a check for understanding by the teacher at this point in the project.

Assignment 4

Using the provided requirement referenced in the RFP for the number of toilets, sinks and drinking fountains per student and the provided cost of each fixture, students will create equations that model the cost of each type of fixture. Students will convert measurements through dimensional analysis. Equations from this assignment will be recorded in the engineering notebook for use in the final report. These calculations allow the students to confirm that their design is compliant with the RFP. The engineering notebook will be used as a check for understanding by the teacher at this point in the project.

Assignment 5

Using the provided requirement for heating and air conditioning capacity per cubic foot of school buildings room volume and given a standard HVAC unit, its airflow rate and the desired time of air replenishment, students determine the size (diameter) of the cylindrical vents needed for these given constraints. Students determine for their final report how much air (volume) will be in their vents at any given time and produce a graph showing the volume of the air in the vents as a function of the vent's radius, which is a cubic function and find the graph's maximum and other key features. The graph and process from this assignment will be recorded in team's engineering notebooks to use for the final report. Students will then create a presentation (PowerPoint, digital media, etc.) to share their design at this point with the class. The audience will provide feedback and ask questions about the design in order for each project team to make considerations for revision. The presentation will be graded by the teacher with a rubric.

Assignment 6

Students will create a spreadsheet containing their equations of their findings throughout this unit that can be used during the next unit regarding design constraints that may have a range of possible values. Students will revise their values to make adjustments with respect to their budget. Students will then revisit their "Need to Knows" through the lens of a new student next year. They will update according to the knowledge they have gained through the analysis process. They will reflect on any mathematical challenges (i.e., persevering, problem solving, etc.) they encountered and their new understandings. This reflection can take the form of a written response to a prompt, a journal entry in the engineering notebook, or a digital response by teams and/or individual students.

Unit 3: Design

The goals of Unit 3 are for teams to select the site for their high school and perform the detail design using the results of the data from Units 1 and 2. Site selections will be based on survey data and analysis of sites in the previous units. From this decision, students will begin making decisions regarding the design of their high school. Using graphs, functions and equations to make those decisions, students will create the third section on Design of their culminating report with mathematical evidence of their process. Design elements include the structures, plumbing, electrical, lighting, Heating, Ventilation and Air Conditioning (HVAC), and Information Technology (IT) considerations.

Unit 3: Assignments

Assignment 1

Students will select a site for their high school based on the student population size and features of the school. First they will define the amount of square footage they need for their high school and will be provided three different geometrically shaped site options. Groups will be given a survey of each site which contains the angles of elevation. They will use trigonometric ratios and solve problems involving right triangles to determine the height and angles of the land. Students will manipulate geometric area and volume formulas to determine the length, area and volume of all sites. Groups will be given gradient costs (i.e., grading cost will be $3.61 per square foot) and sample costs for building one-story vs multi-story buildings. Given time constraints for this course, the teacher may address some top-level building codes (i.e., height limitations, maximum parking allowance, etc.). Each group will select the best site on which to build their high school based on their calculations and on choices already made regarding costs and desired features. Expanding upon their work with trigonometric ratios and right triangles from the land plot work, they will apply those same mathematical concepts to create a sketch of what their high school would look like within that site. They will create a document that shows a 2-dimensional rough sketch of a single-story high school model on their chosen site and that defines the process they went through to decide on that site for their high school.

Assignment 2

After determining what their school will look like within their chosen site, students will make final determinations Re: the structural design of their high school. They will decide if it will be single-story, 2-story, etc., or a combination. Other considerations will be: building materials, windows, room for growth, and dimensions of rooms and structures. Students will use linear equations to model room perimeters and resulting quadratic equations to model room areas, and perform arithmetic operations on polynomials to calculate the areas and volumes of the buildings. For the product of Assignment 2, groups will revise and/or add on to the sketch from Assignment 1 to show the 3-dimensional aspects of the building (including key features, dimensions, etc.). They can cut up the pieces of the original sketch, use CAD, etc.

Assignment 3

Students will identify which buildings and rooms will require plumbing. They will use the guideline of 11 gallons per day per student to decide the total length of pipes needed to be able to flow water throughout the campus. Groups will use flow rate equations given by the teacher which they will manipulate to calculate the pressure required to service their design at a given flow rate per unit.Students read and interpret graphs of pressure vs flow rate from manufacturers to size their pipe requirements. For the product of Assignment 3, teams will write a log of their mathematical process for selecting the pipes in their engineering notebooks. This description can include text, calculations and/or visuals and will be used as a check for understanding by the teacher at this point in the project. At this point, students will create a mock report to the client (the school district) containing a summary and plans of their design so far. The report will include an informative description of all elements of the high school that have been designed, a projection of what is being considered for the final design, and a cost write-up of the entire project. The report will be turned in to the teacher to be read as the "client." The teacher will provide feedback for the project team to consider in future revisions.

Assignment 4

Next, students will identify commercial building voltage levels and estimate how much current and power will be required for the school. The unit circle will be used to understand how and why a sine function behaves the way it does, and to understand how AC power is generated. Students will graph AC power as a periodic sine wave to understand the phenomena generated by a unit circle and to determine the period and the amplitude from the graph they create. They will use radical and absolute value functions to describe and graph the root mean square (RMS) of the AC power function. After creating the graph, they will write a paragraph explaining the electrical capacity for the building using the RMS values to justify their reasoning. This paragraph will be included in their engineering notebooks and final report under the analysis section.

Assignment 5

Project groups will decide what types of information technology they will incorporate into their high school. The decisions will include whether students will use laptops or desktops and an audio/video plan (cameras, monitors, sound systems, webcasting capability, etc.) for the school. They will research or be given information regarding bandwidth needed to support computer labs and desktops in classrooms vs laptops and/or tablets for individual students. Students will model the wireless router link budget in decibels as an exponential function to estimate the coverage area of each router. From all of this data, they will report the type, location and total number of wireless routers required in the school. This information will be included in their engineering notebook to be reviewed by the teacher.

Assignment 6

Project Team groups will generate a draft of the third section (Design) of their culminating report to present to the school district. The goal of the Design section of the report is to show all of the mathematical calculations and analysis used to determine the design considerations of the high school design project. Evidence must be included from the

survey data, structural data, plumbing analysis, electrical analysis, HVAC analysis and IT decisions made in Assignments 1-5. Drafts of the third section are used to answer: Which site was chosen to build on? Why? What does the structure of the high school look like? What will be the design and cost of the plumbing, electrical, lighting, HVAC and IT contained in the high school? What was the process to make all of these decision, including mathematical data and analysis? Once the third section is drafted, project teams will pair up to share their engineering notebooks and project designs with each other. The sharing will be structured with discourse questions and responses that are designed to give project teams constructive feedback that they can use moving forward. After team pairs share together, there will be a whole-class share out and discussion of the project designs. The purpose of the discussion will be to bring to light any concerns or issues that teams may need to consider as they make revisions in their designs.

Unit 4: Plan Build

Project teams will create a scale drawing of their high school using the six trigonometric ratios and the criteria of parallel and perpendicular lines to conform to specific building codes for walkways, parking lot space, etc. Students will either hand draw, use CAD, or some other appropriate technology. The plan drawing will include the shapes of three dimensional objects as a cross section in two-dimensions. Also included are the specifications for each subsystem, the requirements from the RFP versus the design capabilities. Finally, students will receive feedback from surveys about their school plan and make any adjustments needed.

Unit 4: Assignments

Assignment 1

Groups will make confirming calculations using trigonometric ratios through decomposing shapes into triangles (i.e., parking and building areas and measurements). Students will solve problems involving right triangles regarding building code requirements such as ramps for handicap access, walkways,etc.Students will utilize quadratic functions to create area models of different features of the blueprint (such as parking lot size and shape, walkway widths and locations). Teams will hand draw, use Sketchup Pro, or use another available CAD system to create a scale drawing of their high school. The scale drawing may be included in the engineering notebook or can be a separate product for review by the teacher.

Assignment 2

Teams will analyze all data from Unit 3, including survey data, structural design, plumbing, electrical, lighting, HVAC and IT data and summarize it into a specifications document for the school, including all of the subsystems. They will reason quantitatively (MP 2), attend to precision (MP 6), look for and make sense of structure (MP 7), and look for and express regularity in repeated reasoning (MP 8) during the analysis process. The specifications document will include sections for each assignment in Unit 3 with specific

design selections for each subsystem (i.e., type of light bulbs, size of plumbing pipes, number of wireless routers, etc.).

Assignment 3

Teams will review the RFP requirements and determine how they met these requirements within their design. They will construct a viable argument (MP3) for their design capabilities using text and mathematical evidence and prepare an RFP requirements versus design capabilities table document (compliance matrix). This matrix will demonstrate specifically that their design complies with all of the requirements. Students will use appropriate tools strategically (MP 5) and include supporting calculations and graphs as quantifiable backup data for their design. The calculations and graphs come from previous assignments, e.g., the linear equations they used in Unit 3, Assignment 2 to model room perimeters.

Assignment 4

Students will create a survey to send to a statistically significant sample of their student population to gather feedback on their general design and features. The survey will include text explaining the general design and features, and will include both quantitative and qualitative feedback options. After receiving feedback, the teams will revisit their design and make revisions. During revisions, they will use mathematical data from the survey (e.g., inferences about population parameters, deciding if their model is consistent with the results from the data-generating process) and previous calculations (e.g., linear equations, trigonometric ratios) to decide on their edits. This editing process will be recorded in their engineering notebooks.

Assignment 5

Project Team groups will generate a draft of the final section of their culminating report. The goal of the final section is to show all of the mathematical calculations and analysis used to determine the build considerations of the high school design project. Evidence must be included from the scale drawing, specifications report and requirements of RFP vs capabilities from Assignments 1-3. Drafts of the final section are used to answer: What mathematical data was used to create the scale drawing of the high school? What are the specifications for each subsystem and how were those decisions made? How are the RFP requirements being met within this design? What are the quantifiable backup data used to substantiate these decisions?

Unit 5: Validation

The final unit of the comprehensive engineering-based course will culminate in a presentation and report of the high school design. Students will present on the process, including their research and analysis, mathematical models and design solutions. The final report will include final drafts of all sections written in Units 1-4. An optional component of the final unit could be a cumulative math exam which would include all Common Core Algebra 2/Trig standards and the set of Geometry standards included in traditional Algebra 2/Trig courses. As an option, students could review all the math used

within the course to take a cumulative exam on all Advanced Algebra standards, including the Geometry standards that are included in traditional Algebra 2/Trig courses. This unit could be a good point in the course to begin review for Smarter Balanced as students will be reviewing all of their work from the year and putting it together for the final project.

Unit 5: Assignments

Assignment 1: Presentation

Teams will create a presentation for the school districts to persuade them to choose their design. The presentation can be digital (PSA, PowerPoint, Prezi, etc.) or on poster board. The presentation should include: the major points from their requirements, analysis, design and build portions of the engineering design process. Presentations will also include explicit information regarding the Mathematical Practices used during the project, e.g., Make sense of problems and persevere in solving them (MP 1). School board members or other community members will be invited to attend the presentations. Community members will ask questions and provide feedback to the groups.

Assignment 2: Final Report

Students will compile sections of the final report written in Units 1-4 and make revisions as necessary based on the feedback from their presentation. This final report should include four major components: requirements, analysis, design and build. These are the four elements of the engineering design process. Students will reference calculations, such as functions used for the AC power and trigonometric ratios for the building considerations, within the design process as evidence to support their design choices. Revisions should also include recalculating costs, revisiting mathematical models and any other mathematical calculations used which informed the revision. Algebra 2 topics that will be reviewed as part of this final report include: linear equations and functions, quadratic equations and functions, polynomials, rational and radical functions, exponential and logarithmic functions, statistics and probability and trigonometry. Teams will submit the final report to school districts for consideration.

Optional Assignment 3: Scale Model

Students apply all the math practices and standards within their course to build a scale model of their high school design (using 3D software, 3D printer, foam board, etc.) Students will use their data analysis from the graphs made earlier to justify their plans. These models can be displayed with student presentation and reports and can even be included in a new round of presenting to an authentic audience.

Texts and Resources:

Algebra 2/Trig district-approved textbook/curriculum

Engineering Design Process district-approved curriculum

Websites:
http://daerospace.com/ for design components of hydraulic systems and mechanical systems

http://www.werc.org/assets/1/assetmanager/rfpwritingguide.pdf for guidance in writing a Request for Proposal (RFP)

http://www.corestandards.org/Math/ for specific Common Core Math Standards and Mathematical Practices descriptions

http://www.sciencebuddies.org/engineering-design-process/engineering-design-process-steps.shtml for more information on the Engineering Design Process

http://www.sciencebuddies.org/engineering-design-process/engineering-design-notebook.shtml for how to implement Engineering Design Notebooks into your classroom instruction and assessment

http://www.usgbc.org/leed for more information on Leadership in Energy and Environmental Design