Title: Geometry by Design!

Length of Course: Full Year (2 semesters; 3 trimesters; 4 quarters)

Subject Area – Discipline: Mathematics ("c") – Geometry

CTE Sector: Building and Construction Trades

CTE Pathway: Cabinetry, Millwork and Woodworking

Grade/Level(s): 9-11

Overview:

This is an integrated course utilizing Geometry concepts in a Building and Construction Trades environment. The course is designed to prepare students for the natural progression to higher math courses, through a course rich in connections to construction projects that will generate interest in the math and increase students' likelihood of success. The applications throughout the course allow students to see the connection between mathematical concepts and the construction of a scale or full-size dwelling. Also, this course could be part of a Building Trades and Construction Academy as an intermediate course in a sequence of construction and mathematical courses. Through the comprehensive integration of woodworking and geometry, students discover geometric concepts as they design and build woodworking products. This course is designed for students who enjoy learning mathematics through creativity and hands-on projects. "Geometry by Design!" clearly articulates connections between geometric concepts and the creation of 3-dimensional wooden masterpieces.

Course Content:

Unit 1 - Introduction: Get in Shape and Stay Safe

Students receive instruction and extended practice in measurement and marking/layout, the fundamental skills which will be needed to complete all of the geometry and woodworking units and assignments that follow in this course. These concepts are also a good launching platform for this integrated course as they are, by themselves, common to both geometry and woodworking curriculum. Students practice the key concepts of general shop safety: no running in the shop, no horse play, and never distract the the operator of any given piece of machinery. Students learn the specific safety rules for the tooling that is applicable to the task at hand and acquire the knowledge and skills required to work in a safe environment. Students learn the vocabulary of geometry as it relates to the cabinet making and woodworking industries. They study math and building sequences related to measurements, geometry, and practical building applications. Students use specific geometry tools such as a ruler,

protractor, and compass as well as various measuring and marking tools specific to all unit projects, such as tape measures and squares. Students measure and lay-out components of basic dimensional shapes on paper as well as create a construction plan for a simple wooden project - a book shelf with triangular supports - then construct those projects from their plans.

Major Topics:

- ★ Review of fractions and decimals: converting fractions to highest or lowest terms, improper fractions and mixed numbers, common denominators, and adding, subtracting, multiplying, and dividing with decimals and fractions.
- ★ Reading a ruler and a tape measure while incorporating fractional measurements to 1/16 of an inch in a building project.
- ★ Introduction to design by creating project drawings labeling all dimensions, stressing the importance of accuracy in woodworking design and mathematical calculations. Definitions of point, line, line segment, ray, plane, angle, and vertex, using geometry discourse to discuss, identify, and lay-out all of these.
- ★ Diameter, radius, and circumference including circular shapes where other circular shapes fit inside of them; applying the area formulas for circles and cylinders. Measurement of angles using a protractor and angle bisectors using a compass
- ★ Shop safety procedures, transporting sharp woodworking tools to prevent injury, and shop etiquette as it applies to cleanliness and safety.
- ★ Safe and appropriate use of basic, non powered hand tools including cross cut saws, rip saws, pullsaws, coping saws, hammers and chisels, hand planes, sandpaper of various grits, etc.
- ★ Creation of a plan for a constructed project using dimensional measurements; transferring plans and calculations to a physical project.
- ★ Calculating the area of triangles and quadrilaterals. -The Pythagorean Theorem in theory and application through construction of woodworking projects, such as right triangle shelf-supports.
- ★ Calculating concepts of measuring volume in woodworking, using the volumetric unit "board foot."
- ★ Calculating wood density (specific gravity) to identify the wood's hardness and species. Analyzing strengths, weaknesses, and resulting functional uses of hardwoods and softwoods.
- ★ Calculating lumber moisture content and the effects in the process of drying

Resources: The Woodworker's Complete Shop Reference (*Popular Woodworking*), by Jennifer Churchill (Chapter 4: "Dimensions: Furniture Design and Measuring Devices")
Wood Technology & Processes, (Text and Workbook), by Mark D. Feirer and John L. Feirer, 2011.

<u>Unit 1 Key Assignments</u>

Safety First Critical Thinking Logic Chains: Students develop their reasoning ability by first writing hypotheses and conclusions regarding shop and tool safety. They'll further develop their use of logic by creating logical-chains (both humorous and serious) as used

in geometric proofs, describing the cause-and-effect of following the rule (or not following the rule) as preparation for their general shop safety and machine tool safety tests. 100% accuracy is mandatory on all safety tests.

Design and Build a Bookshelf: Before starting construction, students create scale drawings for their projects, labeling all dimensions. From the scale drawings, students produce an orthographic projection. The scale drawings and orthographic projections are checked for accuracy before construction begins. Students determine the moisture content and densities of various blocks of wood to identify the species and its properties such as hardness, density, effects of moisture content and drying. Students then measure, layout and build a rectangular bookshelf with triangular support corbels, utilizing one board foot of lumber and various non-powered hand tools. Students calculate area, perimeter, volume, and linear measurement. Students learn to apply measurements to a dimensional object to have it fit a specific area.

Unit 2 - Parallel Lines: The Beauty of Parallel Lines

After this strong foundation in measuring and marking, students investigate parallel, perpendicular, and transversal lines, recognize their importance in the woodworking environment, and identify which major woodworking tools create them. Conjectures are developed relating to co-linear points, parallel and perpendicular lines, parallel lines cut by a transversal, and the relationships between the various angles that are created. These conjectures are examined and proven true or false with the appropriate theorems being developed. At the same time, students develop foundational woodworking skills with portable power tools and stationary power tools that will be used safely and appropriately throughout the remainder of the course, including the table saw, the band saw, and the router. Students apply these geometric and woodworking skills by designing and creating a cutting board with a transversal inlay.

Major Topics:

- ★ Logical chains (syllogisms) and cause-and-effect rationale in geometry and in woodworking, particularly in regards to safety.
- ★ Calculation of corresponding-, interior-, exterior-, same-side-, and opposite-angles created by a transversal that intersects with parallel lines.
- ★ Testing conjectures; formal proofs; axioms, postulates, and theorems.
- ★ Constructing parallel lines with compass and straightedge on paper and on wood using the jointer, table saw, and clamping lamination techniques.
- ★ Safety procedures for rotating woodworking machinery to prevent damage to equipment and self. -Installation and tuning of a band saw blade, ensuring the upper and lower band saw wheels are coplanar.
- ★ Creating a transverse groove across a laminated woodworking project and calculating expected angles, using either a table saw or router for an inlay.
- ★ Form versus function: using appropriate finishing techniques for safe food handling.

Resources: Wood Technology & Processes, (Text and Workbook), by Mark D. Feirer and John L. Feirer, 2011.

Unit 2 Key Assignments

Design and Build a Cutting Board with Transversal Inlay: Students explore coplanar parallel lines and transversals by designing and building a cutting board made of laminated strips with a transversal inlay (dado cut) to demonstrate their knowledge of parallel lines and the relationships between corresponding-, interior-, exterior-, same-side-, and alternate angles. The design with all angle calculations is approved before construction begins. Students joint, rip, glue, clamp, route, sand, and apply finish to complete the project. The size of individual cutting-boards (and hence the angle of the transversal) will vary, resulting in each student developing a unique plan of expected measures and an individual demonstration of accuracy.

Unit 3 - Polygons: The Shape of Construction

Having mastered the basics of measurement and lines, as well as the safe use of woodworking tools, students will create geometric shapes that are useful in cabinetmaking and wood product design. Students develop conjectures about parallel lines and transversals, quadrilaterals and regular polygons, and then demonstrate their understanding of polygons through the design and building of a hexagonal-topped plant stand. Students also design and build a rectangular frame project (for a mirror or picture) demonstrating their knowledge of mitering, angles, speed square, sanding and finishing work. Students continue to demonstrate the safe and appropriate use of applied woodworking and construction techniques with power tools including all previously used tools, as well as the miter saw.

Major Topics:

- ★ Definitions of convex, concave, and regular polygons. -Definitions of the tangent, sine, and cosine ratios and the inverse trigonometric functions
- ★ Spatial proof of the Pythagorean theorem: using the Pythagorean theorem to calculate missing information of right triangles and its converse to ensure squareness.
- ★ Calculation of the area of 2D polygons and composite shapes.
- ★ Determining trigonometric ratios by taking measurements of appropriate sides of right triangles and by using the Unit Circle. -Using inverse tangent functions to determine expected angle measures of a project's diagonals.
- ★ Using orthographic projections to create construction plans for all woodworking projects
- ★ Special triangle relationships (30o-60o-90o, 45o-45o-90o, 3-4-5) and how they apply to the woodworking industry such as a squaring a cabinet door or mirror frame
- ★ Use of appropriate woodworking tools to cut or sand a given side of the polygon
- ★ Proper approaches to rotating woodworking machinery to prevent damage to equipment and self.
- ★ Introduction of advanced tools, such as the miter saw, and their safe and appropriate usage.
- ★ Finding volume and density of wood in various polygonal shapes to determine wood

species.

- ★ Choosing appropriate woodworking tools and techniques to create miters, chamfers and bevels
- ★ Defining central, interior, and exterior angles and formulas for miters, chamfers and bevels.

Resources: Wood Technology & Processes, (Text and Workbook), by Mark D. Feirer and John L. Feirer, 2011.

Unit 3 Key Assignments

Design and Build a Polygonal-topped Plant Stand: Students create an orthographic representation (design) of a hexagon-topped plant stand including all necessary calculations of interior and exterior angles. Students build the frame of the table with individually laminated pieces of wood, making it necessary to accurately cut each one to the proper angle. Students use a miter saw, or a table saw with a clamping miter gauge, for cutting the regular angles to the desired measures.

Design and Build a Rectangular Mirror or Picture Frame: Students determine the relationships between trigonometric ratios and right triangle angle measures. Using learned joinery methods of the cabinetmaking trade, students design and create a rectangular framed project such as a mirror or picture frame. They use the Pythagorean theorem to determine the expected length of the diagonals to ensure the frame is square. Students use inverse trigonometric ratios to predict angles formed by diagonals in the design and final product.

Unit 4 - Congruence: Creating Congruent Shapes

Having dealt with measurement, lines, and geometric shapes, students focus on the properties of those shapes. Students investigate congruence, developing an understanding that congruent means exactly the same: something that is identical or reproduced. From a mathematical perspective, students understand the concept of congruence as being the same, and from a woodworking perspective, students create components that are the same. Students design and build a drawer box with a checkerboard top to illustrate congruence. Students investigate other shapes and discover what information is necessary to prove them congruent.They design and build triangular frames where the design of the frame indicates known and not known angles. Students are then introduced to tessellations and the three ways tessellations can be created: reflection, rotation, and translation. They apply their knowledge of tessellations to the design and creation of an object with congruent elements: a wooden puzzle consisting of congruent puzzle pieces. The Scroll saw is also now introduced, as it is the appropriate tool of choice for creating puzzle pieces.

Major Topics:

- ★ Congruency--creating congruent shapes and proving triangle congruence (SSS, ASA, SAS).
- ★ Geometric symbols for lines, rays, segments.

- ★ Tessellations: three types and how each is created.
- ★ Design and layout using orthographic projections for visualizing a woodworking project as well as creating a material estimation and cut list.
- ★ Maintaining a clean shop environment for the health and safety of the students.
- ★ Transporting sharp woodworking tools to prevent injury to self and others.
- ★ Introduction of progressively advanced tools and their safe and appropriate usage. Appropriate tool selection and usage for both design and woodworking aspects.

Resources: Online tesselation construction:
http://www.shodor.org/interactivate/activities/Tessellate/
Wood Technology & Processes, (Text and Workbook), by Mark D. Feirer and John L. Feirer, 2011.

Unit 4 Key Assignments

Checking Congruence (Squares): In pairs, students use a variety of woodworking machines and joinery techniques to construct an enclosed drawer box with a checkerboard top of congruent squares. Students use the principles of parallel and perpendicular lines to ensure accuracy in the design. All relevant and necessary calculations are given on the plan. Students laminate cross-sectional pieces of lumber to create a project that demonstrates the properties of parallel and perpendicular lines with individual pieces of congruent lumber that come together to make a whole project.

Proving Triangle Congruence (Triangles): Students investigate what information (or corresponding parts) of triangles is necessary to prove that triangles are congruent. Students investigate SSS, SAS, ASA, AAS, and HL as possible methods and also disprove AAA, ASS, and SSA. Students build portions of triangular frames based on known angles and sides. If an angle is given, a mitered joint is glued together to show it cannot vary. If an angle is not given, a hinge is used to show that the angle can vary. If a side length is given, the length is cut and assembled. If the side length is not given, the student will determine the proper length of that side (if possible). In the case of SSA, students physically find the different triangles that can be made given the angle and sides specified.

Puzzling Congruence (Complex Design): Students investigate and create three different tessellations: a reflection, a rotation, and a translation. In groups, students create a tessellation using familiar shapes that resemble puzzle pieces. Students design and layout the wooden puzzle pieces then generate a working model using the scroll saw. Upon completion of this project, students have developed artistic skills of creating tessellations, understand the geometry behind them, and are able to use a scroll saw to create congruent pieces with wood. Unit 5: Scaling Up

Unit 5 - Similarity: Scaling Up

Following the focus on congruence and similarity, students now address similarity and scale drawings. Students explore the scale factor by enlarging objects to discover the concept of similarity. Students first study proportion and scale to enlarge or shrink line

segments, followed by studying proportion and scale in basic shapes such as triangles, quadrilaterals, and other polygons. Students then build an octagonal treasure chest with an inlaid, rabbeted lid, to illustrate the similarity between it and the frame. Students select an everyday object (e.g, a toothbrush, a smartphone, a puzzle piece) to enlarge by a specific scale factor (percentage), attending to its intricate details using wood and other materials. Students use an architect or engineering scale to measure for the enlargement. Students study how changes in dimensions affect surface area and volume and make arguments about these changes using ratios and exponents. Portable power saws such as the saber saw are now introduced, as they are very useful in cutting out these larger projects. When building the enlargement, students will demonstrate proper tool selection in their woodworking solution.

Major Topics:

- ★ Scale Factor: using ratios to understand enlarging and reducing size while maintaining shape.
- ★ Applying uniform scale factor to an object.
- ★ How surface area, volume, and board footage are affected by the scale factor and using the proper ratios and exponents to calculate those relationships.
- ★ Ratios and Proportion as applied to adjusting which pulleys the belt runs on in a drill press, to adjust the drill presses RPMs.
- ★ Proper approach to rotating woodworking machinery to prevent damage to equipment and self.
- ★ Safety procedures will be demonstrated on specific tools that are applicable to the chosen project of the student.

Unit 5 Key Assignments

Supersize Me!: After studying scale factor, students choose an everyday object and enlarge it using a feasible scale factor of their choice and appropriate layout tools in the shop. Students use wood to design all or most components of the object but may supplement the object with other materials. For example, students could create â€œSoma Puzzlesâ€ of varying sizes, a four-foot toothbrush, or large iPod. Students apply their knowledge of ratio and proportion to create the enlarged objects. After designing the object, students analyze how increasing dimension affects surface area and volume. The outcome of this project is a new, larger object. Students understand “scaling up” and “scaling down” in theory and in practice.

Treasure Chest: To bring together the concepts of congruence and similarity, students combine past project experiences to design and create an octagon-shaped treasure chest with a rabbeted lid. The design phase will include all calculations of angles and other necessary measurements. The rectangles that form the side of the chest will be congruent, and the inner part of the octagon lid will be similar to the outer part. Upon completing this assignment, students realize that similarity and congruence are everywhere and know how to apply the concepts to creating real-word objects more concretely, at the end of this project, students have created a treasure chest.

Unit 6 - Exploring 3D Polyhedra: Adding Dimension

Now that students have a strong understanding of 2-dimensional objects, they move to the study of 3- dimensional objects. Students study and develop formulas for rectangular prisms, pyramids, cylinders, cones, and spheres as well as the relationships of these objects to base polygons. Students create isometric representations and build all of these figures with wood, with a goal of creating equivalent volumes. Students are introduced to the lathe by creating a cylinder of a specified volume. Students design and build a spinning top to investigate the geometric properties of a cone, and to further develop their skills, tapering on a lathe. Students compose and decompose these 3D figures to find the surface area and volume. Students will use appropriate woodworking tools and techniques to design and build the aforementioned figures.

Major Topics:

- ★ Developing formulas for volume and surface area of rectangular prisms, pyramids, cylinders, and cones.
- ★ Using formulas to calculate board feet and finishing materials required for woodworking projects (such as a pyramid, cylinder, cone, or sphere) constructed to match the volume of a given object of a different shape.
- ★ The impact of a change in linear dimensions on area and volume measurements and the related material usage for woodworking project costs.
- ★ Composing and decomposing shapes by adding or subtracting polyhedrons
- ★ Creating nets (orthographic projections) of 3D figures

Resources: 3D visualization of polyhedrons and their nets:
http://www.uff.br/cdme/pdp/pdp-html/pdp-en.html

Unit 6 Key Assignments

Build a Cylinder and a Spinning Top: In order to introduce formulas for finding surface area and volume of 3-dimensional objects and in preparation for unit 7, students will be introduced to the lathe by creating a cylinder of a specific volume, and then by designing and building a beginner's spindle project, a spinning top. They will learn to use calipers to measure the diameter of round workpieces. For the top, the finished shape is to be a cone, and the volume of wood as well as surface area on this finished product will be measured and calculated. Students will measure the volume of their project using sand displacement to derive the formula for the volume of a cone based on the formula for a cylinder.

Build a Polyhedra: Students design and build a polyhedra with an exact volume. Students further develop their understanding of volume by selecting and designing a 3-dimensional polyhedron, such as a pyramid, cone, or sphere, and building it with the appropriate woodworking tools, to match the volume of the teacher's model, which is a regular prism.

Unit 7 - Planes, Space, and Coordinated Circles: Concepts in Segmented Vessels!

Planes, Space, and Coordinated Circles: Concepts in Segmented Vessels! Students now begin their study of how geometric circles and arcs join the 3-dimensional action. Students study the five platonic solids, their truncated figures, various stellated figures,

and then choose one to build, using contrasting species of wood, to emphasize the different 2-dimensional shapes that come together to form the stellated figure or platonic solid. Through the design and construction of segmented vessels such as platonic solids, stellated figures, geodesic spheres, segmented bowls, or truncated icosahedrons (soccer balls), students explore and study relationships among segments on chords, secants, and arcs. Students calculate formulas and write equations for circles, calculating radii and diameter. Students calculate and use inscribed and circumscribed circles around regular polygons to ensure proper placement of segments in their design. Students investigate the surface area and volume of the vessels constructed. They design, measure, and cut all segments using tools such as the table saw and miter saw. Students ensure all segmented portions that are to be glued and clamped together are coplaner by using hand planes, portable or stationary sanders, or stationary planer machines. Students use faceplate bowl-turning tools and techniques to shape and hollow vessels as desired.

Major Topics:

- ★ Inscribed and circumscribed circles: inscribed angle theorem.
- ★ Determination of area, circumference, chords, secants, arcs, tangents and transversals.
- ★ Platonic solids and Stellated figures.
- ★ Calculations of radii and diameters for gluing portions together concentrically and for mounting work on the lathe.
- ★ Calculating the volume of the finished vessel and sectional planes
- ★ Calculating the board feet consumed vs. usage in final project
- ★ Safe and appropriate use of woodworking tools and machines used in creating segmented vessels (3D objects made of several joined pieces)
- ★ Compound cuts incorporating miters and bevels in 3-D
- ★ Calculating rotation speed (RPMs) at a given circumference to achieve appropriate linear cutting speed
- ★ Woodturning skills, tools, and techniques necessary to design and build a segmented wooden vessel

Resources: http://www.Segmentedwoodturners.org and http://www.lumberjocks.com for segmented vessel inspiration and ideas.

Unit 7 Key Assignments

Build a Segmented Vessel on the lathe: Students develop formulas for the area and circumference of a circle. They use proportional reasoning to develop a method to find the area of a sector and the length of an arc. Based on section views of 3- dimensional objects, and using tools including compound miter saw, table saw, planers, and the lathe, students design and build segmented wooden vessels (such as a segmented bowl, platter, vase, goblet, or lamp) using shapes such as triangles, circles, trapezoids, and various polygons. Students use relationships to find the length of chords, the measures of inscribed angles, and arcs in the vessel. Students calculate the radii, diameters, and rotation speed at a given circumference for linear cutting speed. Students use faceplate bowl-turning skills, tools, and techniques to shape and hollow

vessels as desired. Students calculate the board feet consumed vs. the board feet (volume) in the finished vessel. Students will use the vessel created to determine formulas for planes, volume, and surface area.

Build a Platonic Solid or Stellated Polyhedra: Students study the meaning and properties of the five platonic solids, their truncated figures, and other various stellated polyhedra. Students learn which compound angles reside between the segments used to compose these figures. Students then choose to build either the third, fourth, or fifth platonic solid, any truncated platonic solid, or a Stellated figure with at least eight sides, by cutting and gluing the miters and bevels of each segment to fit, using their knowledge of the geometric capabilities of each tool to determine their woodworking process.

Texts:

Title: State-Approved Geometry Textbook
Usage: Primary Text, Read in entirety or near entirety

Title: *Wood Technology & Processes (Text and Workbook)*

Publisher: McGraw-Hill
Author(s): Mark D. Feirer and John L. Feirer
Usage: Primary Text, Read in entirety or near entirety

Supplemental Materials:

http://www.woodshopteachers.org
Geogebra: http://www.geogebra.org/cms/
eHow: Tools & Equipment: http://www.ehow.com/ehow_home-tools-and-equipment/
OSHA Workplace Safety Powerpoints: http://www.freeoshainfo.com/ppt.html
Safety Videos: http://video.wwgoa.com/tips http://powertoolinstitue.com
GoogleSketchUp: 3D Modeling for Everyone: http://sketchup.google.com/
Measurement: http://www.rickyspears.com/rulergame/
http://www.mathplayground.com/measuingangles.html
Segmented Woodturning: http://www.segmentedwoodturners.org/